Optimal Transport Regularized Black Hole Movie Reconstruction

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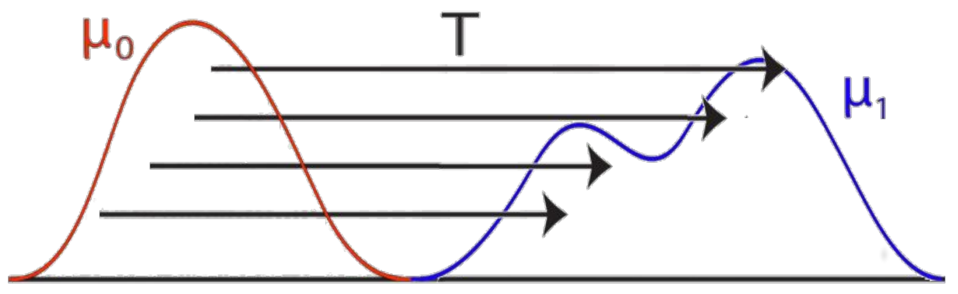
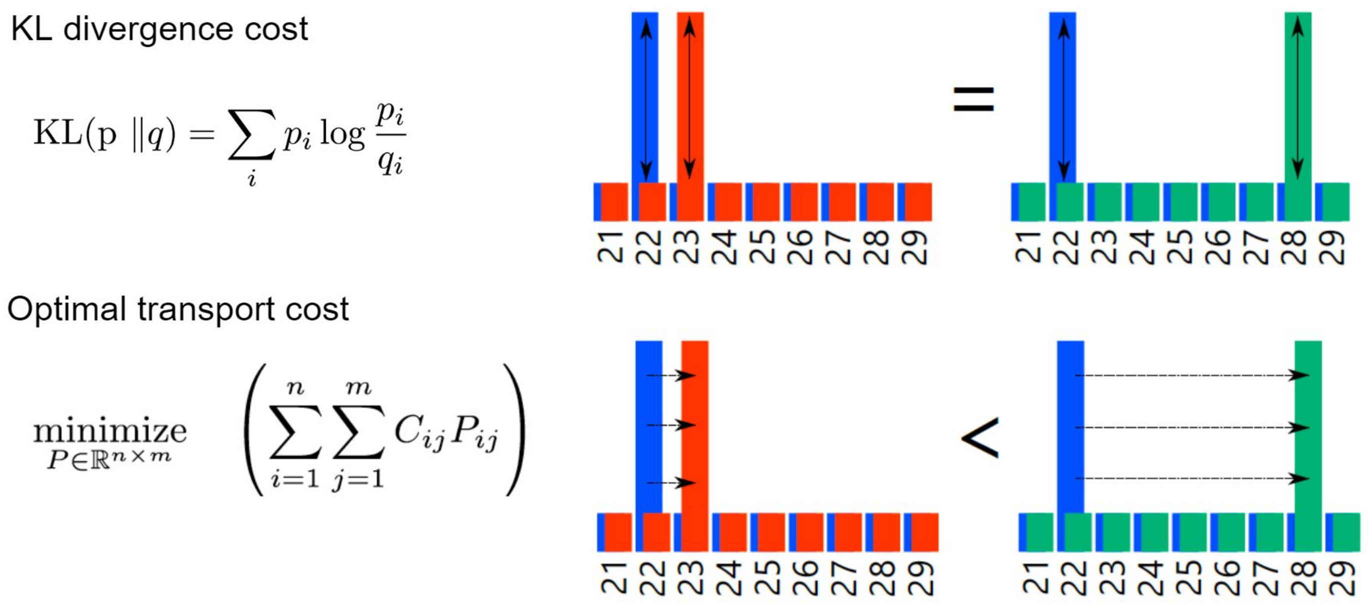
Optimal Transport

Optimal Transport (OT) defines a distance as the least amount of work to transport mass between two probability distributions given a cost function.



Unlike pixel-based distance measures, the Wasserstein/OT distance contains information about the underlying domain of the distribution. In recent years, the Wasserstein distance has become increasingly discussed in various aspects of computer science. Since it provides a notion of distance between distributions, it can be used in machine learning as an objective function[1,2], in image interpolation as a way to calculate geodesics between images[3], and in many other applications to computer graphics and vision[4].



Mathematical formulation:

Standard OT Cost   Entropy term for convergence

$$\underset{P\in\mathbb{R}^{n\times m}}{\text{minimize}} \left(\sum_{i=1}^{n}\sum_{j=1}^{m} C_{ij}P_{ij}\right) - \varepsilon \sum_{i=1}^{n}\sum_{j=1}^{m} P_{ij}(1-\log P_{ij})$$

Cost matrix C: distance between pixels i and j

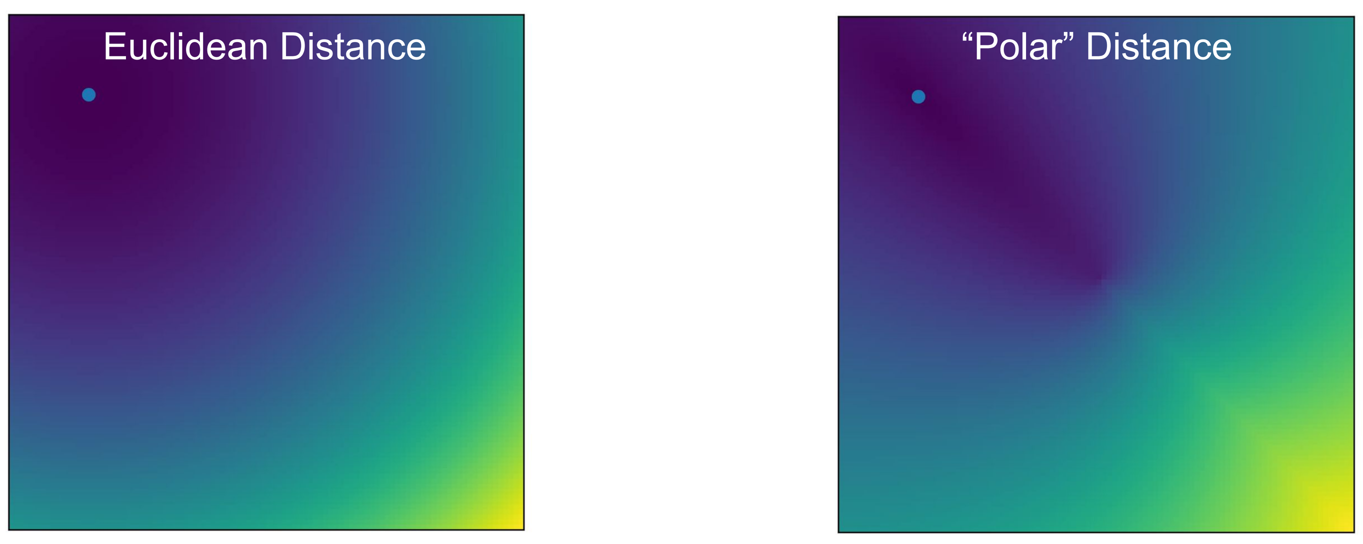
Transportation matrix P: how much mass to move from pixel i to j

[1] Frogner et al. (2015) “Learning with a Wasserstein Loss.” NIPS
[2] Arjovksy et al. (2017) “Wasserstein Generative Adversarial Networks.” PMLR
[3] Hug et al. (2015) “Multi-physics optimal transportation and image interpolation.” ESAIM
[4] Bonneel and Digne (2023) “A survey of Optimal Transport for Computer Graphics and Computer Vision. Computer Graphics Forum

Cost Matrix

Typically, the cost matrix for OT uses the standard Euclidean distance (left), but other options are also possible, such as a “polar” distance that punishes angular displacement more than radial displacement (right).

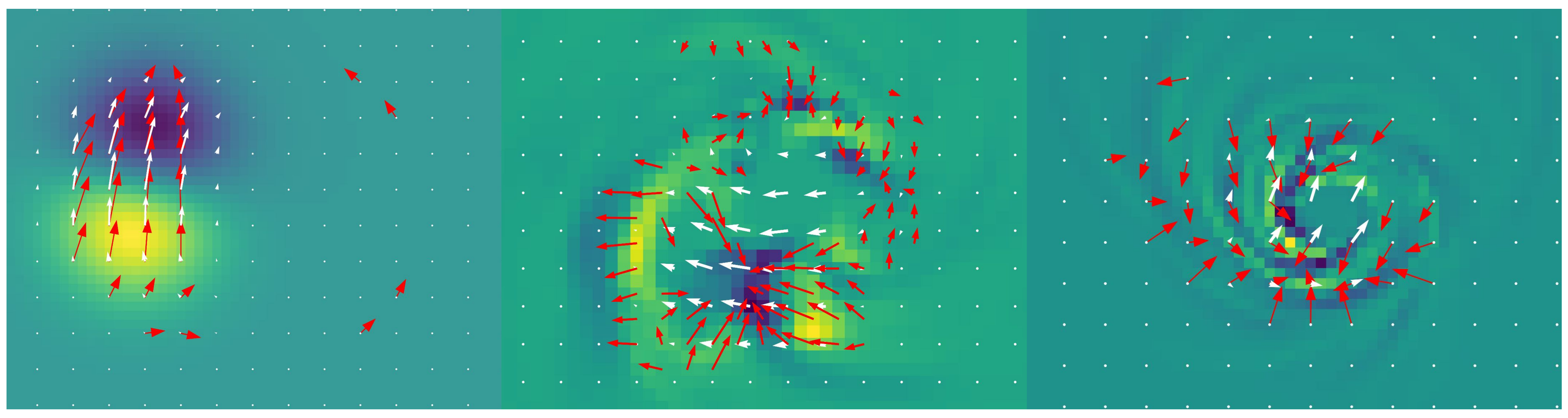
$$d((r_1,\theta_1),(r_2,\theta_2)) = \sqrt{r_1 r_2(\theta_1-\theta_2)^2 + \eta^2(r_1-r_2)^2}$$



Cost to transport mass from the blue pixel

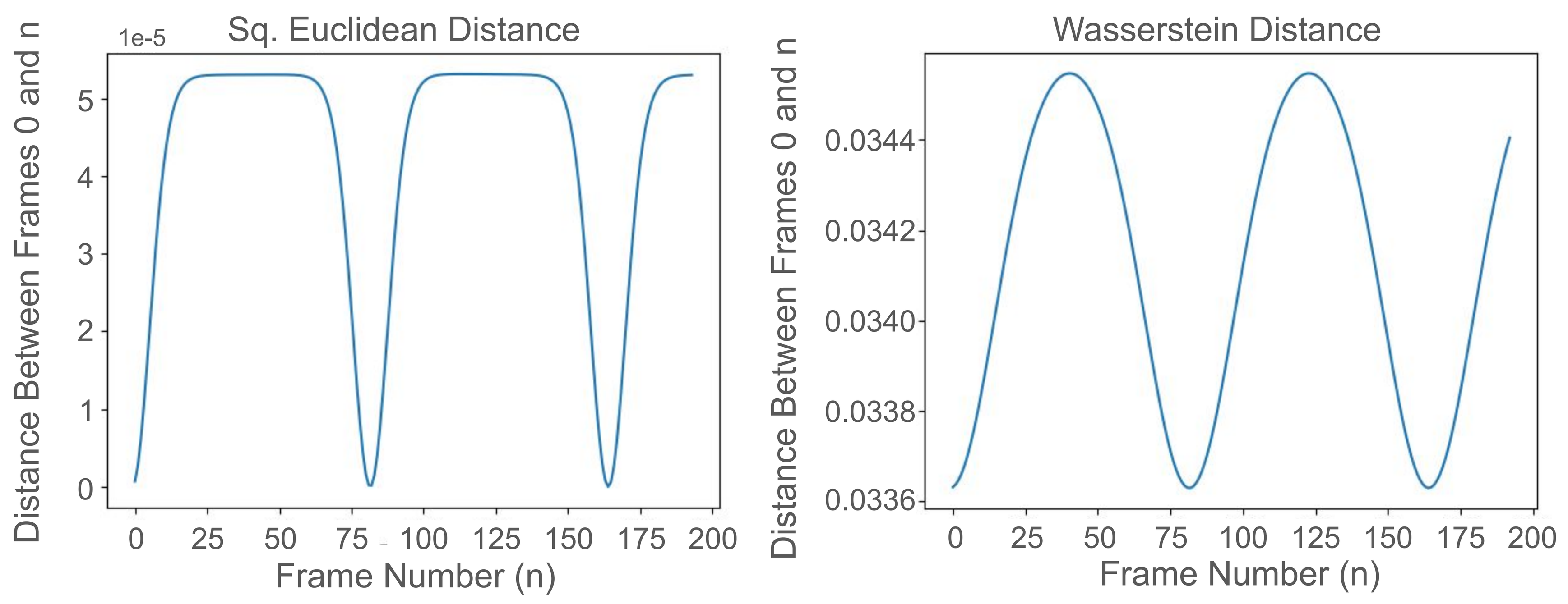
Motivation

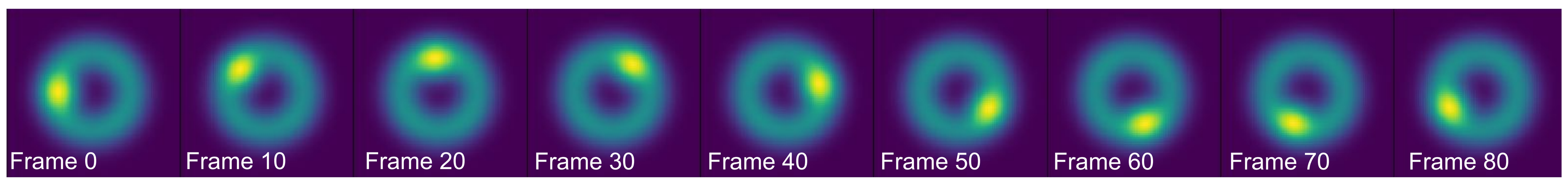
The Wasserstein distance inherently encodes information about underlying physical motion between images through the transportation matrix. Similar to the transportation matrix is the optical flow, which calculates a velocity at points in an image based on pixel intensity changes.

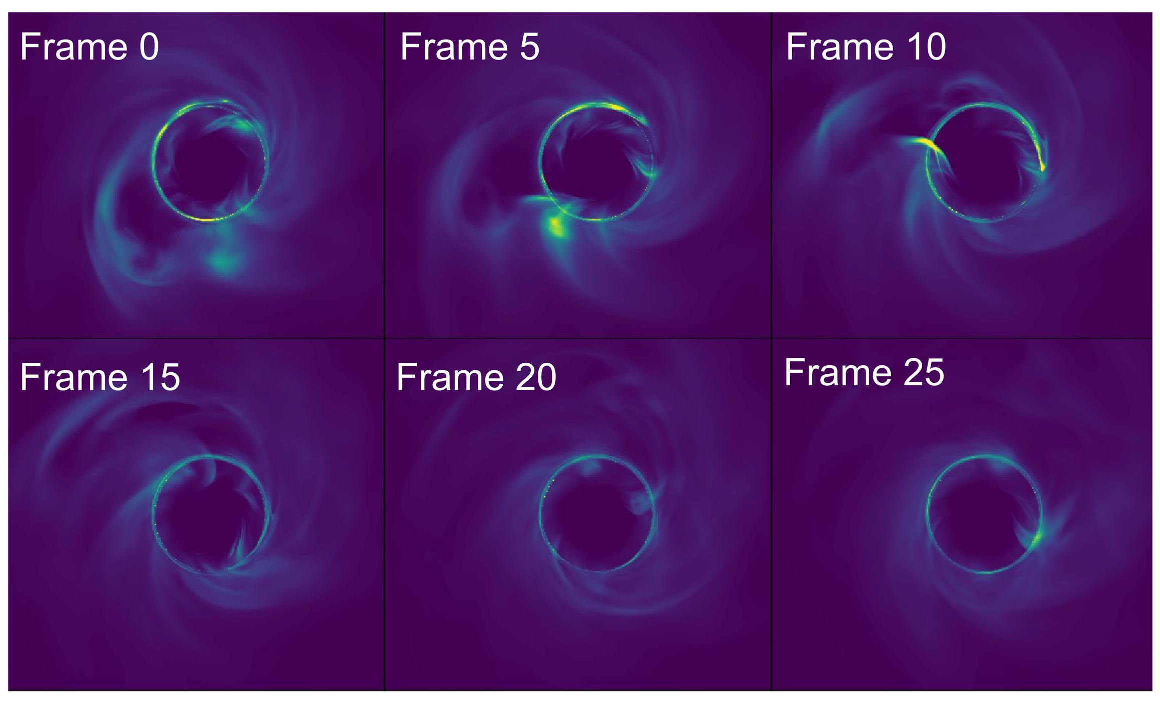


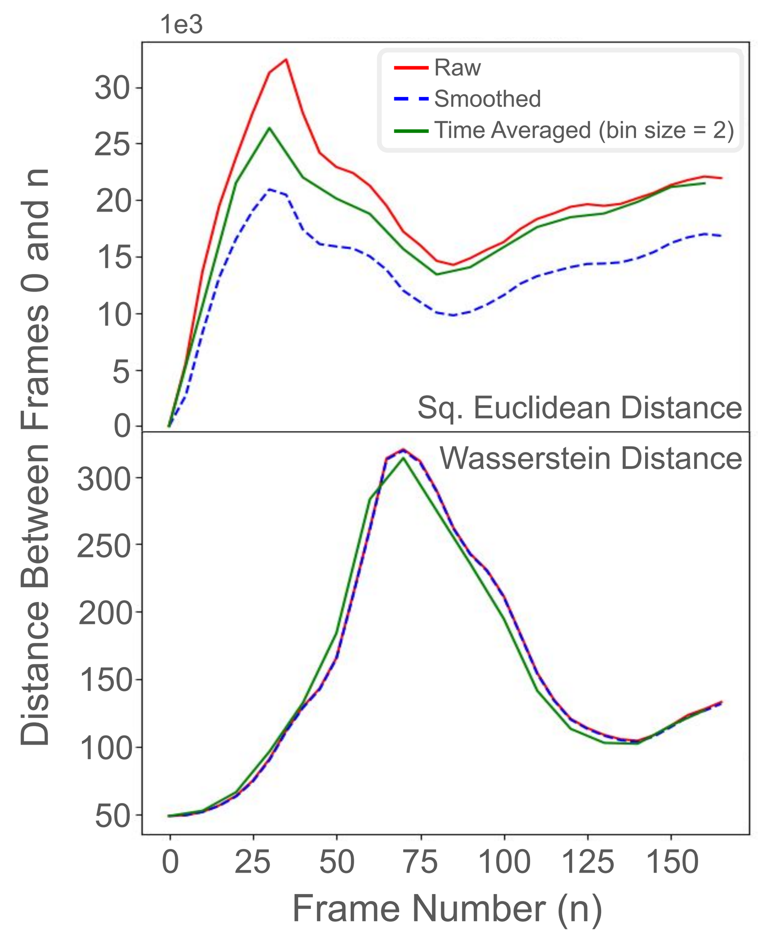
Comparison of OT transportation matrix and optical flow matrix between two frames of a ring and rotating hot spot model (left), a GRMHD model (center), and a jet model (right). The background image shows the difference between the initial and final frames. Bright spots are brighter in the initial frame and dark spots are brighter in the final frame. The white arrows show the optical flow vectors, and the red arrows show the OT transportation matrix, pointing to the “center of mass” of how each pixel’s initial frame flux is distributed in the final frame.

The Wasserstein distance traces motion between frames, allowing for a reduced frame time interval in movie reconstruction. An ideal dynamic regularizer scales with how “different” two frames are; for a rotating model, the regularizer should be maximized at half the rotational period and minimized at the period. The Wasserstein distance does this better than a pixel-wise distance for both the rotating hot spot model (top) and the GRMHD model (bottom).



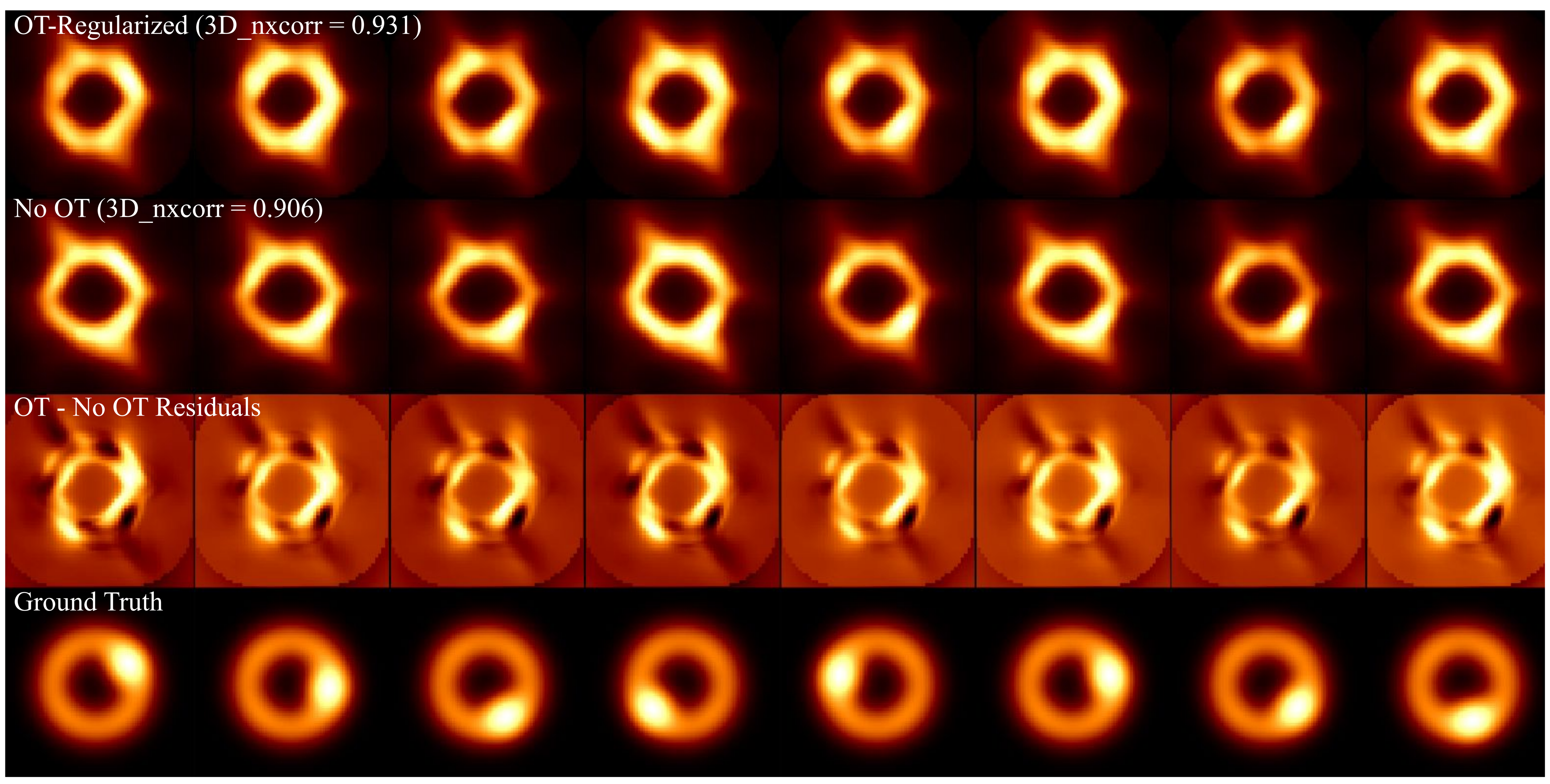




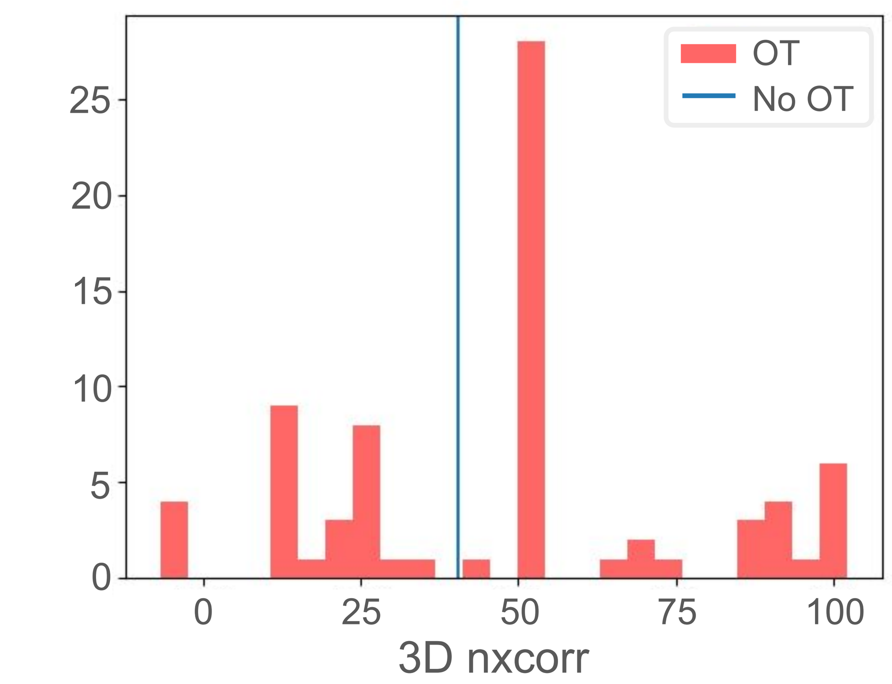


The Wasserstein distance also appears to largely independent of resolution (smoothing with a Gaussian kernel) and time-binning, whereas the pixel-based square Euclidean distance is not.

OT Movie Reconstructions



Comparison of the top OT-regularized movie reconstruction and the top non-OT reconstruction for the ring and rotating hot spot model. The residuals between the two show evidence of a bright area matching the position of the hot spot in the ground truth movie, indicating an improvement in the reconstruction.



Histogram of 3D normalized cross-correlation values (closer to 1 = more similar to ground truth) for all OT reconstructions compared to the non-OT reconstruction with the highest value. Adding OT as a regularizer shows quantitative improvement in the majority of reconstructions.

The OT Wasserstein distance shows promise as a regularizer for coherent movie reconstruction of interferometric data. With the advent of upcoming next-generation VLBI arrays, reconstruction of black hole movies will help further understanding of astrophysics near the event horizon. This is an ongoing project in the EHT Collaboration dynamical imaging group, and we are actively performing further tests on BHEX and ngEHT simulated observations to confirm the viability of the OT distance as a dynamical regularizer.